ECE 162A
Optional HW
Due in Class Tuesday, Dec. 2

1. Many-particle Systems: Symmetry

For this problem, build your solutions as linear combinations of terms like

$$\psi_A(1)\psi_B(2)...\left|\uparrow,\downarrow,...\right\rangle$$

$$where\ A\ and\ B\ are\ energy\ levels;\ 1,2,...\ are\ particle\ labels, and$$

$$\left|\uparrow,\downarrow,...\right\rangle\ means\ that\ paricle\ \#1\ has\ spin\ up,\ particle\ \#2\ has\ spin\ down,...$$

(i) Consider a system of two *independent* electrons which occupy the same (spatial) energy level. Write down the form of the spatial, the spin and the overall wave-function. Show your reasoning at every step. Note that ***electrons are Fermions***.

(ii) Write down the form of the spatial, the spin and the overall wave-function for a system of two independent electrons which occupy two distinct spatial energy levels A and B. How many ways can this be done? Show each.

Hint: For (ii), please see Page 309, Eisberg and Resnick, to get started.

2. Exchange Energy

Consider a system of two electrons which occupy two distinct energy levels, namely ***E_1 and E_2***.

(i) Write down the two-particle Hamiltonian, ignoring electron-electron interaction.

(ii) Write the spatial form of the solution exactly as you did for 2(ii). You must get two solutions for the spatial component. Normalize them.

(iii) Determine the expectation value of the energy for each 2-particle wave-function you got in (ii).

(iv) Now we will drop the assumption that the two electrons are independent, and add a coulomb repulsion term to the two-electron Hamiltonian. Write down the resultant two-particle Hamiltonian.

(v) Assuming that the wave-functions will retain their form, find the expectation value of the energy **<*E*>** of the two-particle system with coulomb repulsion included. Simplify the expression as much as possible, knowing that ***E_1 and E_2*** are two of the energy eigen-values for each single-particle Hamiltonian. (Please don't try to evaluate the integrals involving the Coulomb term!)

(vi) In the final form of the expression for **<*E*>**, identify the classical Coulomb repulsion term and the exchange-interaction term. State which solution of 3(ii) will have the higher energy.